1. QUANTUM CHROMODYNAMICS

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1.1. Basics

Quantum Chromodynamics (QCD), the gauge field theory that describes the strong interactions of colored quarks and gluons, is the SU(3) component of the SU(3)×SU(2)×U(1) Standard Model of Particle Physics.

The Lagrangian of QCD is given by

$$\mathcal{L} = \sum_q \bar{\psi}_{q,a} \left(i\gamma^\mu \partial_\mu \delta_{ab} - g_s \gamma^\mu t^C_{ab} \mathcal{A}^C_\mu - m_q \delta_{ab} \right) \psi_{q,b} - \frac{1}{4} F^A_{\mu\nu} F^{A\,\mu\nu} , \tag{1.1}$$

where repeated indices are summed over. The γ^μ are the Dirac γ-matrices. The $\psi_{q,a}$ are quark-field spinors for a quark of flavor q and mass m_q, with a color-index a that runs from $a = 1$ to $N_c = 3$, *i.e.* quarks come in three "colors." Quarks are said to be in the fundamental representation of the SU(3) color group.

The $\mathcal{A}^C_\mu$ correspond to the gluon fields, with C running from 1 to $N_c^2 - 1 = 8$, *i.e.* there are eight kinds of gluon. Gluons are said to be in the adjoint representation of the SU(3) color group. The t^C_{ab} correspond to eight 3×3 matrices and are the generators of the SU(3) group. They encode the fact that a gluon's interaction with a quark rotates the quark's color in SU(3) space. The quantity g_s is the QCD coupling constant. Finally, the field tensor $F^A_{\mu\nu}$ is given by

$$F^A_{\mu\nu} = \partial_\mu \mathcal{A}^A_\nu - \partial_\nu \mathcal{A}^A_\mu - g_s\, f_{ABC} \mathcal{A}^B_\mu \mathcal{A}^C_\nu \qquad \left[t^A, t^B \right] = i f_{ABC} t^C , \tag{1.2}$$

where the f_{ABC} are the structure constants of the SU(3) group.

Neither quarks nor gluons are observed as free particles. Hadrons are color-singlet (*i.e.* color-neutral) combinations of quarks, anti-quarks, and gluons.

Ab-initio predictive methods for QCD include lattice gauge theory and perturbative expansions in the coupling. The Feynman rules of QCD involve a quark-antiquark-gluon ($q\bar{q}g$) vertex, a 3-gluon vertex (both proportional to g_s), and a 4-gluon vertex (proportional to g_s^2).

Useful color-algebra relations include: $t^A_{ab} t^A_{bc} = C_F \delta_{ac}$, where $C_F \equiv (N_c^2 - 1)/(2N_c) = 4/3$ is the color-factor ("Casimir") associated with gluon emission from a quark; $f^{ACD} f^{BCD} = C_A \delta_{AB}$ where $C_A \equiv N_c = 3$ is the color-factor associated with gluon emission from a gluon; $t^A_{ab} t^B_{ab} = T_R \delta_{AB}$, where $T_R = 1/2$ is the color-factor for a gluon to split to a $q\bar{q}$ pair.

The fundamental parameters of QCD are the coupling g_s (or $\alpha_s = \frac{g_s^2}{4\pi}$) and the quark masses m_q.

1.1.1. ***Running coupling***: In the framework of perturbative QCD (pQCD), predictions for observables are expressed in terms of the renormalized coupling $\alpha_s(\mu_R^2)$, a function of an (unphysical) renormalization scale μ_R. When one takes μ_R close to the scale of the momentum transfer Q in a given process, then $\alpha_s(\mu_R^2 \simeq Q^2)$ is indicative of the effective strength of the strong interaction in that process.

The coupling satisfies the following renormalization group equation (RGE):

$$\mu_R^2 \frac{d\alpha_s}{d\mu_R^2} = \beta(\alpha_s) = -\left(b_0\alpha_s^2 + b_1\alpha_s^3 + b_2\alpha_s^4 + \cdots\right) \tag{1.3}$$

where $b_0 = (11C_A - 4n_f T_R)/(12\pi) = (33 - 2n_f)/(12\pi)$ is the 1-loop beta-function coefficient, and the 2, 3 and 4-loop coefficients are $b_1 = (17C_A^2 - n_f T_R(10C_A + 6C_F))/(24\pi^2) = (153 - 19n_f)/(24\pi^2)$, $b_2 = (2857 - \frac{5033}{9}n_f + \frac{325}{27}n_f^2)/(128\pi^3)$, and $b_3 = ((\frac{149753}{6} + 3564\zeta_3) - (\frac{1078361}{162} + \frac{6508}{27}\zeta_3)n_f + (\frac{50065}{162} + \frac{6472}{81}\zeta_3)n_f^2 + \frac{1093}{729}n_f^3)/(4\pi)^4$ [9,10], the last two specifically in the $\overline{\mathrm{MS}}$ scheme (here $\zeta_3 \simeq 1.2020569$). The minus sign in Eq. (1.3) is the origin of asymptotic freedom, *i.e.* the fact that the strong coupling becomes weak for processes involving large momentum transfers ("hard processes"), $\alpha_s \sim 0.1$ for momentum transfers in the 100 GeV –TeV range.

The β-function coefficients, the b_i, are given for the coupling of an *effective theory* in which n_f of the quark flavors are considered light ($m_q \ll \mu_R$), and in which the remaining heavier quark flavors decouple from the theory. One may relate the coupling for the theory with $n_f + 1$ light flavors to that with n_f flavors through an equation of the form

$$\alpha_s^{(n_f+1)}\left(\mu_R^2\right) = \alpha_s^{(n_f)}\left(\mu_R^2\right)\left(1 + \sum_{n=1}^{\infty}\sum_{\ell=0}^{n} c_{n\ell}\left[\alpha_s^{(n_f)}\left(\mu_R^2\right)\right]^n \ln^\ell \frac{\mu_R^2}{m_h^2}\right), \tag{1.4}$$

where m_h is the mass of the $(n_f+1)^{\text{th}}$ flavor, and the first few $c_{n\ell}$ coefficients are $c_{11} = \frac{1}{6\pi}$, $c_{10} = 0$, $c_{22} = c_{11}^2$, $c_{21} = \frac{19}{24\pi^2}$, and $c_{20} = -\frac{11}{72\pi^2}$ when m_h is the $\overline{\mathrm{MS}}$ mass at scale m_h ($c_{20} = \frac{7}{24\pi^2}$ when m_h is the pole mass — see the review on "Quark Masses"). Terms up to $c_{4\ell}$ are to be found in Refs. 11, 12. Numerically, when one chooses $\mu_R = m_h$, the matching is a small effect, owing to the zero value for the c_{10} coefficient.

Working in an energy range where the number of flavors is constant, a simple exact analytic solution exists for Eq. (1.3) only if one neglects all but the b_0 term, giving $\alpha_s(\mu_R^2) = (b_0 \ln(\mu_R^2/\Lambda^2))^{-1}$. Here Λ is a constant of integration, which corresponds to the scale where the perturbatively-defined coupling would diverge, *i.e.* it is the non-perturbative scale of QCD. A convenient approximate analytic solution to the RGE that includes also the b_1, b_2, and b_3 terms is given by (*e.g.* Ref. 14),

$$\alpha_s\left(\mu_R^2\right) \simeq \frac{1}{b_0 t}\left(1 - \frac{b_1}{b_0^2}\frac{\ln t}{t} + \frac{b_1^2\left(\ln^2 t - \ln t - 1\right) + b_0 b_2}{b_0^4 t^2}\right.$$

$$-\frac{b_1^3\left(\ln^3 t-\frac{5}{2}\ln^2 t-2\ln t+\frac{1}{2}\right)+3b_0b_1b_2\ln t-\frac{1}{2}b_0^2b_3}{b_0^6t^3} \quad t\equiv\ln\frac{\mu_R^2}{\Lambda^2}\,, \tag{1.5}$$

again parametrized in terms of a constant Λ. Note that Eq. (1.5) is one of several possible approximate 4-loop solutions for $\alpha_s(\mu_R^2)$, and that a value for Λ only defines $\alpha_s(\mu_R^2)$ once one knows which particular approximation is being used. An alternative to the use of formulas such as Eq. (1.5) is to solve the RGE exactly, numerically (including the discontinuities, Eq. (1.4), at flavor thresholds). In such cases the quantity Λ is not defined at all. For these reasons, in determinations of the coupling, it has become standard practice to quote the value of α_s at a given scale (typically M_Z) rather than to quote a value for Λ.

The value of the coupling, as well as the exact forms of the b_2, c_{10} (and higher order) coefficients, depend on the renormalization scheme in which the coupling is defined, *i.e.* the convention used to subtract infinities in the context of renormalization. The coefficients given above hold for a coupling defined in the modified minimal subtraction ($\overline{\mathrm{MS}}$) scheme [15], by far the most widely used scheme.

1.3.4. ***Measurements of the strong coupling constant***: For this review it was decided to quote a recent analysis by Bethke [244], which incorporates results with recently improved theoretical predictions and/or experimental precision. The central value is determined as the weighted average of the individual measurements. For the error an overall, a-priori unknown, correlation coefficient is introduced and determined by requiring that the total χ^2 of the combination equals the number of degrees of freedom. The world average quoted in Ref. 244 is

$$\alpha_s\left(M_Z^2\right)=0.1184\pm0.0007\,.$$

It is worth noting that a cross check performed in Ref. 244, consisting in excluding each of the single measurements from the combination, resulted in variations of the central value well below the quoted uncertainty, and in a maximal increase of the combined error up to 0.0012. Most notably, excluding the most precise determination from lattice QCD gives only a marginally different average value. Nevertheless, there remains an apparent and long-standing systematic difference between the results from structure functions and other determinations of similar accuracy. This is evidenced in Fig. 1.1 (left), where the various inputs to this combination, evolved to the Z mass scale, are shown. Fig. 1.1 (right) provides strongest evidence for the correct prediction by QCD of the scale dependence of the strong coupling.

Further discussion and all references may be found in the full *Review.*

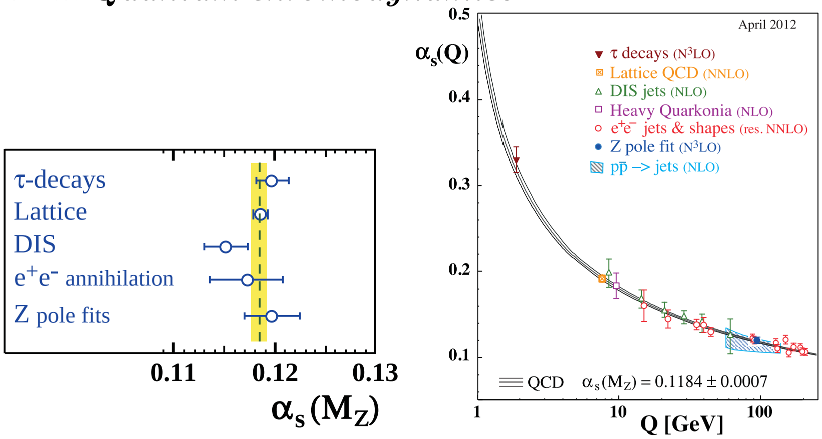

Figure 1.1: Left: Summary of measurements of $\alpha_s(M_Z^2)$, used as input for the world average value; Right: Summary of measurements of α_s as a function of the respective energy scale Q. Both plots are taken from Ref. 244.